9.1 Properties of Radicals

Essential Question

How can you multiply and divide square roots?

EXPLORATION 1 Operations with Square Roots

Work with a partner. For each operation with square roots, compare the results obtained using the two indicated orders of operations. What can you conclude?

a. Square Roots and Addition

Is $\sqrt{36} + \sqrt{64}$ equal to $\sqrt{36 + 64}$?

In general, is $\sqrt{a} + \sqrt{b}$ equal to $\sqrt{a + b}$? Explain your reasoning.

b. Square Roots and Multiplication

Is $\sqrt{4} \cdot \sqrt{9}$ equal to $\sqrt{4 \cdot 9}$?

In general, is $\sqrt{a} \cdot \sqrt{b}$ equal to $\sqrt{a \cdot b}$? Explain your reasoning.

c. Square Roots and Subtraction

Is $\sqrt{64} - \sqrt{36}$ equal to $\sqrt{64 - 36}$?

In general, is $\sqrt{a} - \sqrt{b}$ equal to $\sqrt{a - b}$? Explain your reasoning.

d. Square Roots and Division

Is $\frac{\sqrt{100}}{\sqrt{4}}$ equal to $\sqrt{\frac{100}{4}}$?

In general, is $\frac{\sqrt{a}}{\sqrt{b}}$ equal to $\sqrt{\frac{a}{b}}$? Explain your reasoning.

REASONING ABSTRACTLY

To be proficient in math, you need to recognize and use counterexamples.

EXPLORATION 2 Writing Counterexamples

Work with a partner. A **counterexample** is an example that proves that a general statement is *not* true. For each general statement in Exploration 1 that is not true, write a counterexample different from the example given.

Communicate Your Answer

3. How can you multiply and divide square roots?

4. Give an example of multiplying square roots and an example of dividing square roots that are different from the examples in Exploration 1.

5. Write an algebraic rule for each operation.

a. the product of square roots

b. the quotient of square roots

9.1 Lesson

Core Vocabulary

counterexample, *p. 479*
radical expression, *p. 480*
simplest form of a radical, *p. 480*
rationalizing the denominator, *p. 482*
conjugates, *p. 482*
like radicals, *p. 484*

Previous
radicand
perfect cube

What You Will Learn

- Use properties of radicals to simplify expressions.
- Simplify expressions by rationalizing the denominator.
- Perform operations with radicals.

Using Properties of Radicals

A **radical expression** is an expression that contains a radical. An expression involving a radical with index n is in **simplest form** when these three conditions are met.

- No radicands have perfect nth powers as factors other than 1.
- No radicands contain fractions.
- No radicals appear in the denominator of a fraction.

You can use the property below to simplify radical expressions involving square roots.

Core Concept

Product Property of Square Roots

Words The square root of a product equals the product of the square roots of the factors.

Numbers $\sqrt{9 \cdot 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$

Algebra $\sqrt{ab} = \sqrt{a} \cdot \sqrt{b}$, where $a, b \geq 0$

STUDY TIP

There can be more than one way to factor a radicand. An efficient method is to find the greatest perfect square factor.

EXAMPLE 1 Using the Product Property of Square Roots

a. $\sqrt{108} = \sqrt{36 \cdot 3}$ Factor using the greatest perfect square factor.

$= \sqrt{36} \cdot \sqrt{3}$ Product Property of Square Roots

$= 6\sqrt{3}$ Simplify.

b. $\sqrt{9x^3} = \sqrt{9 \cdot x^2 \cdot x}$ Factor using the greatest perfect square factor.

$= \sqrt{9} \cdot \sqrt{x^2} \cdot \sqrt{x}$ Product Property of Square Roots

$= 3x\sqrt{x}$ Simplify.

STUDY TIP

In this course, whenever a variable appears in the radicand, assume that it has only *nonnegative* values.

Monitoring Progress

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Simplify the expression.

1. $\sqrt{24}$ **2.** $-\sqrt{80}$ **3.** $\sqrt{49x^3}$ **4.** $\sqrt{75n^5}$

Core Concept

Quotient Property of Square Roots

Words The square root of a quotient equals the quotient of the square roots of the numerator and denominator.

Numbers $\sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{\sqrt{4}} = \frac{\sqrt{3}}{2}$

Algebra $\sqrt{\frac{a}{b}} = \frac{\sqrt{a}}{\sqrt{b}}$, where $a \geq 0$ and $b > 0$

EXAMPLE 2 Using the Quotient Property of Square Roots

a. $\sqrt{\frac{15}{64}} = \frac{\sqrt{15}}{\sqrt{64}}$ — Quotient Property of Square Roots

$= \frac{\sqrt{15}}{8}$ — Simplify.

b. $\sqrt{\frac{81}{x^2}} = \frac{\sqrt{81}}{\sqrt{x^2}}$ — Quotient Property of Square Roots

$= \frac{9}{x}$ — Simplify.

You can extend the Product and Quotient Properties of Square Roots to other radicals, such as cube roots. When using these *properties of cube roots*, the radicands may contain negative numbers.

EXAMPLE 3 Using Properties of Cube Roots

STUDY TIP

To write a cube root in simplest form, find factors of the radicand that are perfect cubes.

a. $\sqrt[3]{-128} = \sqrt[3]{-64 \cdot 2}$ — Factor using the greatest perfect cube factor.

$= \sqrt[3]{-64} \cdot \sqrt[3]{2}$ — Product Property of Cube Roots

$= -4\sqrt[3]{2}$ — Simplify.

b. $\sqrt[3]{125x^7} = \sqrt[3]{125 \cdot x^6 \cdot x}$ — Factor using the greatest perfect cube factors.

$= \sqrt[3]{125} \cdot \sqrt[3]{x^6} \cdot \sqrt[3]{x}$ — Product Property of Cube Roots

$= 5x^2\sqrt[3]{x}$ — Simplify.

c. $\sqrt[3]{\frac{y}{216}} = \frac{\sqrt[3]{y}}{\sqrt[3]{216}}$ — Quotient Property of Cube Roots

$= \frac{\sqrt[3]{y}}{6}$ — Simplify.

d. $\sqrt[3]{\frac{8x^4}{27y^3}} = \frac{\sqrt[3]{8x^4}}{\sqrt[3]{27y^3}}$ — Quotient Property of Cube Roots

$= \frac{\sqrt[3]{8 \cdot x^3 \cdot x}}{\sqrt[3]{27 \cdot y^3}}$ — Factor using the greatest perfect cube factors.

$= \frac{\sqrt[3]{8} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x}}{\sqrt[3]{27} \cdot \sqrt[3]{y^3}}$ — Product Property of Cube Roots

$= \frac{2x\sqrt[3]{x}}{3y}$ — Simplify.

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Simplify the expression.

5. $\sqrt{\frac{23}{9}}$ **6.** $-\sqrt{\frac{17}{100}}$ **7.** $\sqrt{\frac{36}{z^2}}$ **8.** $\sqrt{\frac{4x^2}{64}}$

9. $\sqrt[3]{54}$ **10.** $\sqrt[3]{16x^4}$ **11.** $\sqrt[3]{\frac{a}{-27}}$ **12.** $\sqrt[3]{\frac{25c^7d^3}{64}}$

Rationalizing the Denominator

When a radical is in the denominator of a fraction, you can multiply the fraction by an appropriate form of 1 to eliminate the radical from the denominator. This process is called **rationalizing the denominator**.

STUDY TIP

Rationalizing the denominator works because you multiply the numerator and denominator by the same nonzero number a, which is the same as multiplying by $\frac{a}{a}$, or 1.

EXAMPLE 4 Rationalizing the Denominator

a. $\dfrac{\sqrt{5}}{\sqrt{3n}} = \dfrac{\sqrt{5}}{\sqrt{3n}} \cdot \dfrac{\sqrt{3n}}{\sqrt{3n}}$ Multiply by $\dfrac{\sqrt{3n}}{\sqrt{3n}}$.

$= \dfrac{\sqrt{15n}}{\sqrt{9n^2}}$ Product Property of Square Roots

$= \dfrac{\sqrt{15n}}{\sqrt{9} \cdot \sqrt{n^2}}$ Product Property of Square Roots

$= \dfrac{\sqrt{15n}}{3n}$ Simplify.

b. $\dfrac{2}{\sqrt[3]{9}} = \dfrac{2}{\sqrt[3]{9}} \cdot \dfrac{\sqrt[3]{3}}{\sqrt[3]{3}}$ Multiply by $\dfrac{\sqrt[3]{3}}{\sqrt[3]{3}}$.

$= \dfrac{2\sqrt[3]{3}}{\sqrt[3]{27}}$ Product Property of Cube Roots

$= \dfrac{2\sqrt[3]{3}}{3}$ Simplify.

The binomials $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$, where a, b, c, and d are rational numbers, are called **conjugates**. You can use conjugates to simplify radical expressions that contain a sum or difference involving square roots in the denominator.

LOOKING FOR STRUCTURE

Notice that the product of two conjugates $a\sqrt{b} + c\sqrt{d}$ and $a\sqrt{b} - c\sqrt{d}$ does not contain a radical and is a *rational* number.

$(a\sqrt{b} + c\sqrt{d})(a\sqrt{b} - c\sqrt{d})$
$= (a\sqrt{b})^2 - (c\sqrt{d})^2$
$= a^2b - c^2d$

EXAMPLE 5 Rationalizing the Denominator Using Conjugates

Simplify $\dfrac{7}{2 - \sqrt{3}}$.

SOLUTION

$\dfrac{7}{2 - \sqrt{3}} = \dfrac{7}{2 - \sqrt{3}} \cdot \dfrac{2 + \sqrt{3}}{2 + \sqrt{3}}$ The conjugate of $2 - \sqrt{3}$ is $2 + \sqrt{3}$.

$= \dfrac{7(2 + \sqrt{3})}{2^2 - (\sqrt{3})^2}$ Sum and difference pattern

$= \dfrac{14 + 7\sqrt{3}}{1}$ Simplify.

$= 14 + 7\sqrt{3}$ Simplify.

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Simplify the expression.

13. $\dfrac{1}{\sqrt{5}}$ **14.** $\dfrac{\sqrt{10}}{\sqrt{3}}$ **15.** $\dfrac{7}{\sqrt{2x}}$ **16.** $\sqrt{\dfrac{2y^2}{3}}$

17. $\dfrac{5}{\sqrt[3]{32}}$ **18.** $\dfrac{8}{1 + \sqrt{3}}$ **19.** $\dfrac{\sqrt{13}}{\sqrt{5} - 2}$ **20.** $\dfrac{12}{\sqrt{2} + \sqrt{7}}$

EXAMPLE 6 Solving a Real-Life Problem

The distance d (in miles) that you can see to the horizon with your eye level h feet above the water is given by $d = \sqrt{\frac{3h}{2}}$. How far can you see when your eye level is 5 feet above the water?

SOLUTION

$$d = \sqrt{\frac{3(5)}{2}}$$ Substitute 5 for h.

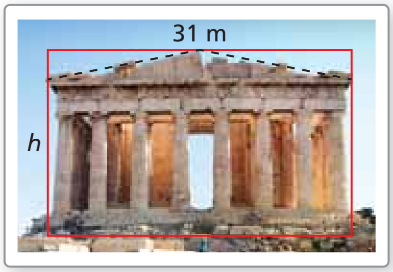
$$= \frac{\sqrt{15}}{\sqrt{2}}$$ Quotient Property of Square Roots

$$= \frac{\sqrt{15}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$$ Multiply by $\frac{\sqrt{2}}{\sqrt{2}}$.

$$= \frac{\sqrt{30}}{2}$$ Simplify.

▶ You can see $\frac{\sqrt{30}}{2}$, or about 2.74 miles.

EXAMPLE 7 Modeling with Mathematics



The ratio of the length to the width of a *golden rectangle* is $(1 + \sqrt{5}) : 2$. The dimensions of the face of the Parthenon in Greece form a golden rectangle. What is the height h of the Parthenon?

SOLUTION

1. **Understand the Problem** Think of the length and height of the Parthenon as the length and width of a golden rectangle. The length of the rectangular face is 31 meters. You know the ratio of the length to the height. Find the height h.

2. **Make a Plan** Use the ratio $(1 + \sqrt{5}) : 2$ to write a proportion and solve for h.

3. **Solve the Problem**

$$\frac{1 + \sqrt{5}}{2} = \frac{31}{h}$$ Write a proportion.

$$h(1 + \sqrt{5}) = 62$$ Cross Products Property

$$h = \frac{62}{1 + \sqrt{5}}$$ Divide each side by $1 + \sqrt{5}$.

$$h = \frac{62}{1 + \sqrt{5}} \cdot \frac{1 - \sqrt{5}}{1 - \sqrt{5}}$$ Multiply the numerator and denominator by the conjugate.

$$h = \frac{62 - 62\sqrt{5}}{-4}$$ Simplify.

$$h \approx 19.16$$ Use a calculator.

▶ The height is about 19 meters.

4. **Look Back** $\frac{1 + \sqrt{5}}{2} \approx 1.62$ and $\frac{31}{19.16} \approx 1.62$. So, your answer is reasonable.

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21. **WHAT IF?** In Example 6, how far can you see when your eye level is 35 feet above the water?

22. The dimensions of a dance floor form a golden rectangle. The shorter side of the dance floor is 50 feet. What is the length of the longer side of the dance floor?

Performing Operations with Radicals

Radicals with the same index and radicand are called **like radicals**. You can add and subtract like radicals the same way you combine like terms by using the Distributive Property.

STUDY TIP

Do not assume that radicals with different radicands cannot be added or subtracted. Always check to see whether you can simplify the radicals. In some cases, the radicals will become like radicals.

EXAMPLE 8 Adding and Subtracting Radicals

a. $5\sqrt{7} + \sqrt{11} - 8\sqrt{7} = 5\sqrt{7} - 8\sqrt{7} + \sqrt{11}$ Commutative Property of Addition

$= (5 - 8)\sqrt{7} + \sqrt{11}$ Distributive Property

$= -3\sqrt{7} + \sqrt{11}$ Subtract.

b. $10\sqrt{5} + \sqrt{20} = 10\sqrt{5} + \sqrt{4 \cdot 5}$ Factor using the greatest perfect square factor.

$= 10\sqrt{5} + \sqrt{4} \cdot \sqrt{5}$ Product Property of Square Roots

$= 10\sqrt{5} + 2\sqrt{5}$ Simplify.

$= (10 + 2)\sqrt{5}$ Distributive Property

$= 12\sqrt{5}$ Add.

c. $6\sqrt[3]{x} + 2\sqrt[3]{x} = (6 + 2)\sqrt[3]{x}$ Distributive Property

$= 8\sqrt[3]{x}$ Add.

EXAMPLE 9 Multiplying Radicals

Simplify $\sqrt{5}(\sqrt{3} - \sqrt{75})$.

SOLUTION

Method 1 $\sqrt{5}(\sqrt{3} - \sqrt{75}) = \sqrt{5} \cdot \sqrt{3} - \sqrt{5} \cdot \sqrt{75}$ Distributive Property

$= \sqrt{15} - \sqrt{375}$ Product Property of Square Roots

$= \sqrt{15} - 5\sqrt{15}$ Simplify.

$= (1 - 5)\sqrt{15}$ Distributive Property

$= -4\sqrt{15}$ Subtract.

Method 2 $\sqrt{5}(\sqrt{3} - \sqrt{75}) = \sqrt{5}(\sqrt{3} - 5\sqrt{3})$ Simplify $\sqrt{75}$.

$= \sqrt{5}\left[(1 - 5)\sqrt{3}\right]$ Distributive Property

$= \sqrt{5}(-4\sqrt{3})$ Subtract.

$= -4\sqrt{15}$ Product Property of Square Roots

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Simplify the expression.

23. $3\sqrt{2} - \sqrt{6} + 10\sqrt{2}$

24. $4\sqrt{7} - 6\sqrt{63}$

25. $4\sqrt[3]{5x} - 11\sqrt[3]{5x}$

26. $\sqrt{3}(8\sqrt{2} + 7\sqrt{32})$

27. $(2\sqrt{5} - 4)^2$

28. $\sqrt[3]{-4}(\sqrt[3]{2} - \sqrt[3]{16})$

9.1 Exercises

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Vocabulary and Core Concept Check

1. **COMPLETE THE SENTENCE** The process of eliminating a radical from the denominator of a radical expression is called ________________.

2. **VOCABULARY** What is the conjugate of the binomial $\sqrt{6} + 4$?

3. **WRITING** Are the expressions $\frac{1}{3}\sqrt{2x}$ and $\sqrt{\frac{2x}{9}}$ equivalent? Explain your reasoning.

4. **WHICH ONE DOESN'T BELONG?** Which expression does *not* belong with the other three? Explain your reasoning.

$-\frac{1}{3}\sqrt{6}$	$6\sqrt{3}$	$\frac{1}{6}\sqrt{3}$	$-3\sqrt{3}$

Monitoring Progress and Modeling with Mathematics

In Exercises 5–12, determine whether the expression is in simplest form. If the expression is not in simplest form, explain why.

5. $\sqrt{19}$
6. $\sqrt{\frac{1}{7}}$
7. $\sqrt{48}$
8. $\sqrt{34}$
9. $\frac{5}{\sqrt{2}}$
10. $\frac{3\sqrt{10}}{4}$
11. $\frac{1}{2+\sqrt[3]{2}}$
12. $6 - \sqrt[3]{54}$

In Exercises 13–20, simplify the expression. *(See Example 1.)*

13. $\sqrt{20}$
14. $\sqrt{32}$
15. $\sqrt{128}$
16. $-\sqrt{72}$
17. $\sqrt{125b}$
18. $\sqrt{4x^2}$
19. $-\sqrt{81m^3}$
20. $\sqrt{48n^5}$

In Exercises 21–28, simplify the expression. *(See Example 2.)*

21. $\sqrt{\frac{4}{49}}$
22. $-\sqrt{\frac{7}{81}}$
23. $-\sqrt{\frac{23}{64}}$
24. $\sqrt{\frac{65}{121}}$
25. $\sqrt{\frac{a^3}{49}}$
26. $\sqrt{\frac{144}{k^2}}$
27. $\sqrt{\frac{100}{4x^2}}$
28. $\sqrt{\frac{25v^2}{36}}$

In Exercises 29–36, simplify the expression. *(See Example 3.)*

29. $\sqrt[3]{16}$
30. $\sqrt[3]{-108}$
31. $\sqrt[3]{-64x^5}$
32. $-\sqrt[3]{343n^2}$
33. $\sqrt[3]{\frac{6c}{-125}}$
34. $\sqrt[3]{\frac{8h^4}{27}}$
35. $-\sqrt[3]{\frac{81y^2}{1000x^3}}$
36. $\sqrt[3]{\frac{21}{-64a^3b^6}}$

ERROR ANALYSIS **In Exercises 37 and 38, describe and correct the error in simplifying the expression.**

37. ✗

$$\sqrt{72} = \sqrt{4 \cdot 18} = \sqrt{4} \cdot \sqrt{18} = 2\sqrt{18}$$

38. ✗

$$\sqrt[3]{\frac{128y^3}{125}} = \frac{\sqrt[3]{128y^3}}{125} = \frac{\sqrt[3]{64 \cdot 2 \cdot y^3}}{125} = \frac{\sqrt[3]{64} \cdot \sqrt[3]{2} \cdot \sqrt[3]{y^3}}{125} = \frac{4y\sqrt[3]{2}}{125}$$

In Exercises 39–44, write a factor that you can use to rationalize the denominator of the expression.

39. $\frac{4}{\sqrt{6}}$

40. $\frac{1}{\sqrt{13z}}$

41. $\frac{2}{\sqrt[3]{x^2}}$

42. $\frac{3m}{\sqrt[3]{4}}$

43. $\frac{\sqrt{2}}{\sqrt{5} - 8}$

44. $\frac{5}{\sqrt{3} + \sqrt{7}}$

In Exercises 45–54, simplify the expression. *(See Example 4.)*

45. $\frac{2}{\sqrt{2}}$

46. $\frac{4}{\sqrt{3}}$

47. $\frac{\sqrt{5}}{\sqrt{48}}$

48. $\sqrt{\frac{4}{52}}$

49. $\frac{3}{\sqrt{a}}$

50. $\frac{1}{\sqrt{2x}}$

51. $\sqrt{\frac{3d^2}{5}}$

52. $\frac{\sqrt{8}}{\sqrt{3n^3}}$

53. $\frac{4}{\sqrt[3]{25}}$

54. $\sqrt[3]{\frac{1}{108y^2}}$

In Exercises 55–60, simplify the expression. *(See Example 5.)*

55. $\frac{1}{\sqrt{7} + 1}$

56. $\frac{2}{5 - \sqrt{3}}$

57. $\frac{\sqrt{10}}{7 - \sqrt{2}}$

58. $\frac{\sqrt{5}}{6 + \sqrt{5}}$

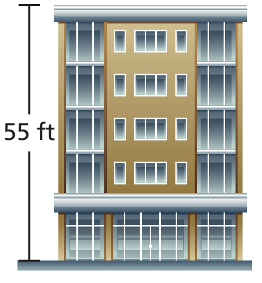
59. $\frac{3}{\sqrt{5} - \sqrt{2}}$

60. $\frac{\sqrt{3}}{\sqrt{7} + \sqrt{3}}$

61. MODELING WITH MATHEMATICS The time t (in seconds) it takes an object to hit the ground is given by $t = \sqrt{\frac{h}{16}}$, where h is the height (in feet) from which the object was dropped. *(See Example 6.)*

a. How long does it take an earring to hit the ground when it falls from the roof of the building?

b. How much sooner does the earring hit the ground when it is dropped from two stories (22 feet) below the roof?



62. MODELING WITH MATHEMATICS The orbital period of a planet is the time it takes the planet to travel around the Sun. You can find the orbital period P (in Earth years) using the formula $P = \sqrt{d^3}$, where d is the average distance (in astronomical units, abbreviated AU) of the planet from the Sun.

a. Simplify the formula.

b. What is Jupiter's orbital period?

63. MODELING WITH MATHEMATICS The electric current I (in amperes) an appliance uses is given by the formula $I = \sqrt{\frac{P}{R}}$, where P is the power (in watts) and R is the resistance (in ohms). Find the current an appliance uses when the power is 147 watts and the resistance is 5 ohms.

64. MODELING WITH MATHEMATICS You can find the average annual interest rate r (in decimal form) of a savings account using the formula $r = \sqrt{\frac{V_2}{V_0}} - 1$, where V_0 is the initial investment and V_2 is the balance of the account after 2 years. Use the formula to compare the savings accounts. In which account would you invest money? Explain.

Account	Initial investment	Balance after 2 years
1	\$275	\$293
2	\$361	\$382
3	\$199	\$214
4	\$254	\$272
5	\$386	\$406

In Exercises 65–68, evaluate the function for the given value of x. Write your answer in simplest form and in decimal form rounded to the nearest hundredth.

65. $h(x) = \sqrt{5x};\ x = 10$

66. $g(x) = \sqrt{3x};\ x = 60$

67. $r(x) = \sqrt{\dfrac{3x}{3x^2 + 6}};\ x = 4$

68. $p(x) = \sqrt{\dfrac{x - 1}{5x}};\ x = 8$

In Exercises 69–72, evaluate the expression when $a = -2$, $b = 8$, and $c = \frac{1}{2}$. Write your answer in simplest form and in decimal form rounded to the nearest hundredth.

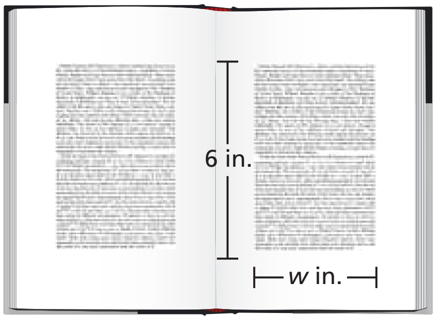
69. $\sqrt{a^2 + bc}$

70. $-\sqrt{4c - 6ab}$

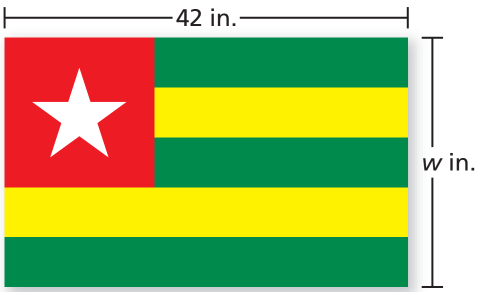
71. $-\sqrt{2a^2 + b^2}$

72. $\sqrt{b^2 - 4ac}$

73. MODELING WITH MATHEMATICS The text in the book shown forms a golden rectangle. What is the width w of the text? *(See Example 7.)*



74. MODELING WITH MATHEMATICS The flag of Togo is approximately the shape of a golden rectangle. What is the width w of the flag?



In Exercises 75–82, simplify the expression. *(See Example 8.)*

75. $\sqrt{3} - 2\sqrt{2} + 6\sqrt{2}$

76. $\sqrt{5} - 5\sqrt{13} - 8\sqrt{5}$

77. $2\sqrt{6} - 5\sqrt{54}$

78. $9\sqrt{32} + \sqrt{2}$

79. $\sqrt{12} + 6\sqrt{3} + 2\sqrt{6}$

80. $3\sqrt{7} - 5\sqrt{14} + 2\sqrt{28}$

81. $\sqrt[3]{-81} + 4\sqrt[3]{3}$

82. $6\sqrt[3]{128t} - 2\sqrt[3]{2t}$

In Exercises 83–90, simplify the expression. *(See Example 9.)*

83. $\sqrt{2}(\sqrt{45} + \sqrt{5})$

84. $\sqrt{3}(\sqrt{72} - 3\sqrt{2})$

85. $\sqrt{5}(2\sqrt{6x} - \sqrt{96x})$

86. $\sqrt{7y}(\sqrt{27y} + 5\sqrt{12y})$

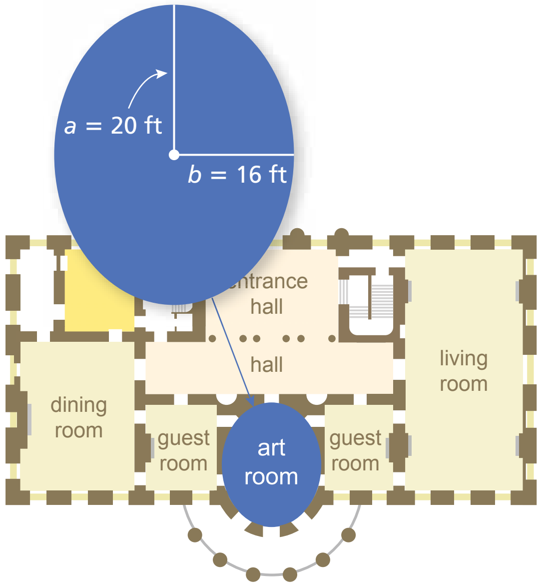
87. $(4\sqrt{2} - \sqrt{98})^2$

88. $(\sqrt{3} + \sqrt{48})(\sqrt{20} - \sqrt{5})$

89. $\sqrt[3]{3}(\sqrt[3]{4} + \sqrt[3]{32})$

90. $\sqrt[3]{2}(\sqrt[3]{135} - 4\sqrt[3]{5})$

91. MODELING WITH MATHEMATICS The circumference C of the art room in a mansion is approximated by the formula $C \approx 2\pi\sqrt{\dfrac{a^2 + b^2}{2}}$. Approximate the circumference of the room.



92. CRITICAL THINKING Determine whether each expression represents a *rational* or an *irrational* number. Justify your answer.

a. $4 + \sqrt{6}$

b. $\dfrac{\sqrt{48}}{\sqrt{3}}$

c. $\dfrac{8}{\sqrt{12}}$

d. $\sqrt{3} + \sqrt{7}$

e. $\dfrac{a}{\sqrt{10} - \sqrt{2}}$, where a is a positive integer

f. $\dfrac{2 + \sqrt{5}}{2b + \sqrt{5b^2}}$, where b is a positive integer

In Exercises 93–98, simplify the expression.

93. $\sqrt[5]{\dfrac{13}{5x^5}}$

94. $\sqrt[4]{\dfrac{10}{81}}$

95. $\sqrt[4]{256y}$

96. $\sqrt[5]{160x^6}$

97. $6\sqrt[4]{9} - \sqrt[5]{9} + 3\sqrt[4]{9}$

98. $\sqrt[5]{2}(\sqrt[4]{7} + \sqrt[5]{16})$

REASONING In Exercises 99 and 100, use the table shown.

	2	$\frac{1}{4}$	0	$\sqrt{3}$	$-\sqrt{3}$	π
2						
$\frac{1}{4}$						
0						
$\sqrt{3}$						
$-\sqrt{3}$						
π						

99. Copy and complete the table by (a) finding each sum $\left(2 + 2, 2 + \frac{1}{4}, \text{etc.}\right)$ and (b) finding each product $\left(2 \cdot 2, 2 \cdot \frac{1}{4}, \text{etc.}\right)$.

100. Use your answers in Exercise 99 to determine whether each statement is *always*, *sometimes*, or *never* true. Justify your answer.

a. The sum of a rational number and a rational number is rational.

b. The sum of a rational number and an irrational number is irrational.

c. The sum of an irrational number and an irrational number is irrational.

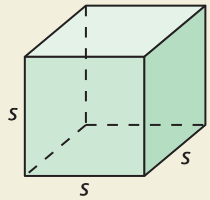
d. The product of a rational number and a rational number is rational.

e. The product of a nonzero rational number and an irrational number is irrational.

f. The product of an irrational number and an irrational number is irrational.

101. **REASONING** Let m be a positive integer. For what values of m will the simplified form of the expression $\sqrt{2^m}$ contain a radical? For what values will it *not* contain a radical? Explain.

102. **HOW DO YOU SEE IT?** The edge length s of a cube is an irrational number, the surface area is an irrational number, and the volume is a rational number. Give a possible value of s.



103. **REASONING** Let a and b be positive numbers. Explain why $\sqrt{ab}$ lies between a and b on a number line. (*Hint:* Let $a < b$ and multiply each side of $a < b$ by a. Then let $a < b$ and multiply each side by b.)

104. **MAKING AN ARGUMENT** Your friend says that you can rationalize the denominator of the expression $\frac{2}{4 + \sqrt[3]{5}}$ by multiplying the numerator and denominator by $4 - \sqrt[3]{5}$. Is your friend correct? Explain.

105. **PROBLEM SOLVING** The ratio of consecutive terms $\frac{a_n}{a_{n-1}}$ in the Fibonacci sequence gets closer and closer to the golden ratio $\frac{1 + \sqrt{5}}{2}$ as n increases. Find the term that precedes 610 in the sequence.

106. **THOUGHT PROVOKING** Use the golden ratio $\frac{1 + \sqrt{5}}{2}$ and the golden ratio conjugate $\frac{1 - \sqrt{5}}{2}$ for each of the following.

a. Show that the golden ratio and golden ratio conjugate are both solutions of $x^2 - x - 1 = 0$.

b. Construct a geometric diagram that has the golden ratio as the length of a part of the diagram.

107. **CRITICAL THINKING** Use the special product pattern $(a + b)(a^2 - ab + b^2) = a^3 + b^3$ to simplify the expression $\frac{2}{\sqrt[3]{x} + 1}$. Explain your reasoning.

Maintaining Mathematical Proficiency

Reviewing what you learned in previous grades and lessons

Graph the linear equation. Identify the x-intercept. *(Section 3.5)*

108. $y = x - 4$ **109.** $y = -2x + 6$ **110.** $y = -\frac{1}{3}x - 1$ **111.** $y = \frac{3}{2}x + 6$

Solve the equation. Check your solution. *(Section 6.5)*

112. $32 = 2^x$ **113.** $27^x = 3^{x-6}$ **114.** $\left(\frac{1}{6}\right)^{2x} = 216^{1-x}$ **115.** $625^x = \left(\frac{1}{25}\right)^{x+2}$